

in many European fields, as well as in America. Some tentative efforts are now being made to introduce the photo-theodolite to India, but the results are hardly mature enough to justify any opinion as to their success. In France photo-topography has been chiefly applied to the field of that which we should term in England "revenue" or "cadastral" survey; and in Canada (a fact which is not recognised by Colonel Laussedat) a still wider opening has been afforded by the Geological Survey, which is practically a small scale topographical survey leading to the first general map of the country. There are, at any rate, records sufficient to enable us to bring the test of actual experience in other countries than France to bear on Colonel Laussedat's estimate of the capabilities of the system. That estimate appears to be absolutely favourable, but it must be contended that the illustrations which support Colonel Laussedat's opinion are not in themselves comprehensive enough to justify the conclusions at which he arrives, which would apparently include all classes of reconnaissance, or survey; in all conditions of ground as suitable for its application.

An official examination into the results of a photo-theodolite survey was conducted in Paris as long ago as the year 1859, and the report of the commissioners nominated by the Academy of Sciences was so favourable that in 1863 a "photo-topographic brigade" was formed, under the direction of Laussedat, which executed surveys on comparatively large scales (from 1/1000 to 1/20000), and which lasted for a period of eight years. The brigade was broken up in 1871, and whilst Colonel Laussedat refrains from commenting on the reasons for its suppression, he clearly indicates that it was for no reason which implied technical failure.

Various modifications of the original system are discussed or recommended, and one or two excellent illustrations of the resulting surveys are given at the end of the book. But it must be noted that the field of survey to which this process has been applied in France is after all but local, and the scale of mapping is comparatively large. For instance, we find in Plate xiii. a reproduction of about 15 square miles of country, originally surveyed on a scale approximating to 12 inches per mile (reduced to one-fourth in reproduction), to which the following details are appended. The survey was completed in ten days in the field, supplemented by two and a half months of subsequent work in the drawing office (bureau). It involved the use of fifty-two photographs, which were taken at thirty-one stations. Of these stations eighteen were stations of triangulation, and the rest "supplementary." The map itself is fully contoured and apparently quite up to the standard, in detail, of maps on a similar scale executed by the English Ordnance Survey. The time (and consequently the expense) involved in its production will of course compare favourably with that of any other known system of surveying; but it would be rash to infer therefrom that photo-topography is under all conditions either a cheap or a rapid method of surveying. In Canada good work has been done by this process on the smaller scales of one inch or two inches per mile, and the system generally is well established. But Canadian surveyors are not prepared to advocate it in entire supersession of the more widely known system of plane

table topography based on triangulation, maintaining that its advantages are confined to comparatively restricted conditions of surface conformation. Thirty-one stations of observation in fifteen square miles of country (giving an average of two "fixings" per square mile) may under certain conditions be sufficient to enable a surveyor to see into the topographical detail of ridge and furrow, plain and gully, that the country presents, and result in a creditable map. But in a vast proportion of the broken and rugged districts presented by the varied physiography of Asia, Africa, or America two stations per mile would certainly not be sufficient, and the accumulation of photographs would rapidly become an unwieldy burden. When we consider the requirements of geographical surveys on yet smaller scales (say 1/500000) it is impossible to concede that the recognised systems of rapid plane tabling in experienced hands, which result in daily outturns which may be reckoned in scores of square miles of finished mapping (no "bureau" work is required by a really well-trained topographer), can be surpassed in rapidity by any more complicated process which has yet been invented.

Possibly the discussion of the application of photography to this most important field of geographical survey may be reserved for a future volume, although it might certainly have been usefully included in the present one. The author is at any rate on perfectly sound ground when he recommends every explorer who makes use of photography for illustrative purposes to fix the position of his views and the direction (or azimuth) of them with careful exactness on his route map; with the assurance that in scientific hands they will prove of immense value in elucidating the topography of the country which they illustrate if they are thus registered.

There is no work in the English language equal to that of Colonel Laussedat as a comprehensive and up-to-date review of the history and development of topography; in the value of its scientific deductions and illustrations; or in the interest which is sustained by the literary skill exhibited. It should find a place in every library of civil or military engineering institutions which professes to maintain an efficient stock of standard works for reference.

T. H. H.

#### EUCLID REVISED.

*Euclid's Elements of Geometry.* Books i.-iv., vi. and xi. By Charles Smith, M.A., and Sophie Bryant, D.Sc. Pp. viii + 460. (London: Macmillan and Co., Ltd., 1901.) Price 4s. 6d.

IF Euclid is to continue as the foundation of geometrical teaching in our schools, this work must be very warmly welcomed. The exact order of Euclid is followed, but (as the editors inform us) with no special regard to the exact words of the translation of Simson (who for a moment becomes "Simpson" in the foot-note on p. 79). There is also a complete absence of the mechanical chopping up of each proposition into separate blocks under the heads of "general enunciation," "particular enunciation," "hypothesis," "construction," "to prove," "proof," "conclusion," which in some textbooks, and in the minds of many boys, has reduced the whole subject to an artificial jargon.

Mrs. Bryant, both as an expert logician and as the daughter of a fellow of Trinity College, Dublin (Rev. W. A. Willock), who had no belief in the appropriateness of Euclid's book except to "grown-up, hard-headed, thinking men," was sure to remove from the path of the young pupil as much of the essential difficulty of Euclid as could be removed consistently with the retention of the book as the basis of school instruction.

To follow the subject in detail, we notice that the editors have deliberately left out alternative proofs of the "Asses' Bridge" on the ground that Euclid's proof is found by experience to be more readily understood than any of the alternative proofs—a statement which surely cannot be well founded. What can be more simple than the proof founded on the superposition of two identical triangles? And, again, if we imagine the bisector of the vertical angle to be drawn, we have the result as a direct consequence of prop. iv. It is not to the point to object that Euclid will not allow us to imagine this bisector unless we can show how to draw it; if the bisector were drawn, the result would follow—that is proof enough. At the end of Book i. there is a large collection of worked-out theorems and problems; and we may specially notice the excellent exposition of the method of analysis and synthesis in pp. 102–106, which will greatly help the pupil who is learning this method of attacking problems. Besides these worked questions, there is a collection of 100 unworked exercises in illustration of Book i.

In Book ii. the fundamental propositions 12 and 13 are proved as an extension of the proposition of Pythagoras (47, Bk. i.) by the famous old windmill figure so familiar to us all; and, as the editors inform us, this proof is found in Lardner's Euclid, but cannot be traced further back. It is strange that the editors of our school Euclids should have overlooked this most interesting and graphic proof. Lardner's Euclid, now seldom seen, is—even compared with the best modern editions—a work of great usefulness and high merit.

There is a note at the end of Book ii. (p. 148) the substance of which is that pure geometry must be kept severely apart from all arithmetical conceptions; and this is followed (p. 150) by a still more remarkable note stating that "in all examinations" the use of + and –, of the abbreviation  $AB^2$  for the square on AB, and of the abbreviation  $AB \cdot BC$  for the rectangle AB, BC, is permitted in writing out all theorems and problems of geometry, provided that these are not given in Euclid's text.

Why such an extraordinary distinction and restriction should exist is incomprehensible to us, and remains so even after we have read the excuse put forward for it by the present editors; and after this excuse comes the statement

"the use of these symbols ought never to be allowed at any time until it is clear that  $AB^2$  and  $AB \cdot BC$  are used by the student simply as the shortest way of writing the square on AB and the rectangle contained by AB and BC, respectively."

Thus the divorce of all arithmetical conception—and, indeed, all quantitative conception—from geometry is advocated; and if the restriction were really carried out both by teachers and by examiners (which it is not), the

teaching of the subject would be rendered much more slow and difficult than it is at present.

Book iii. ends with a very large collection of worked-out questions followed by 100 exercises, a very good feature being the association of each famous result with the name of its discoverer; and a similar remark may be made with regard to Book iv. Book v. is omitted, only the definitions required in Book vi. being given. Euclid's test of proportion—*i.e.* of the equality of the ratio  $A : B$  to the ratio  $C : D$ —is given and applied to six special cases (p. 293) under the heading "Theory of Proportion." This test is, of course, that  $C : D$  will be the same as  $A : B$  if when  $mA \cong nB$  we have  $mC \cong nD$ ; and we wonder whether any beginner in the world is introduced to the notion of the equality of ratios by this means. Probably without a single exception, every boy is first told that  $4 : 2$  is the same as  $6 : 3$ , because 2 is contained in 4 just as often as 3 is contained in 6; and even if the one quantity were not contained an integer number of times in the other, he would be prepared to admit and understand the equality of ratios if this number was an endless decimal, provided it was the same for the two compared ratios. Euclid's test must infallibly be received by the beginner merely as the *ipse dixit* of Euclid; the beginner cannot understand its validity apart from arithmetical notions; and it seems rather grotesque to find it formally employed to prove such a trifle as "magnitudes which have the same ratio to the same magnitude must be equal." Lardner has, as usual, some excellent remarks on this criterion; but his exposition amounts to no justification that could possibly convince the mind of a beginner. Hear also the opinion of the Rev. W. A. Willock on the question ("Elementary Geometry of the Right Line and Circle," p. ix.) :—

"The criterion of proportion used is that of Elrington, by *submultiples*. This test is here adopted because it is more readily understood by young students, and also more conformable to the common notions of proportion. Moreover, it holds good, in all strictness, for commensurable magnitudes; and, as to the incommensurable, it holds equally good if the equisubmultiples taken of the first and third terms be infinitesimals. . . . The right conclusion as to the two tests is, probably, that both should be given in a treatise on elementary geometry, each having its own peculiar advantages."

At the end of Book vi. follows what may be regarded as a small encyclopædia of important results and methods—coaxal circles, harmonic ranges, poles and polars, centres of similitude, inversion, maxima and minima, &c.—an invaluable collection, excellently handled.

Book xi. calls for no detailed remarks: its accompanying illustrations are of the same high order of merit as that which characterises all the special work of the editors.

#### OUR BOOK SHELF.

*The Life-History of British Serpents and their Local Distribution in the British Isles.* By Gerald R. Leighton, M.D. Pp. xvi + 383. 8vo. Illustrated. (London: W. Blackwood and Sons, 1901.) Price 5s. net.

THE idea of supplying the "field-naturalists of the British Isles" with a handbook dealing with the life-history of the native snakes and their distribution is an excellent